Data Mining:

Concepts and Techniques

(3rd ed.)

— Chapter 7 —

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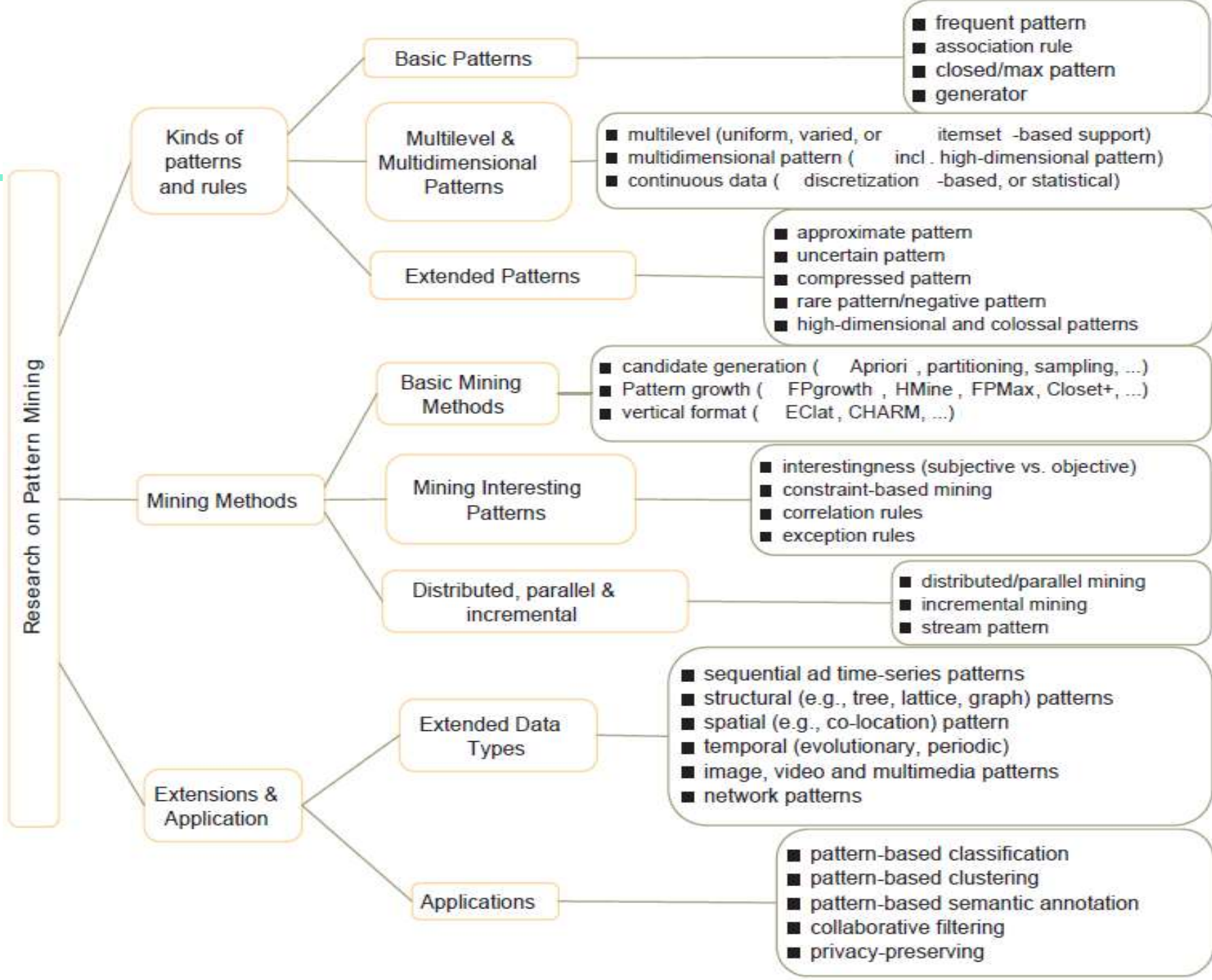
Simon Fraser University

Chapter 7 : Advanced Frequent Pattern Mining

- Pattern Mining: A Road Map

- Pattern Mining in Multi-Level, Multi-Dimensional Space

- Constraint-Based Frequent Pattern Mining

- Mining High-Dimensional Data and Colossal Patterns

- Mining Compressed or Approximate Patterns

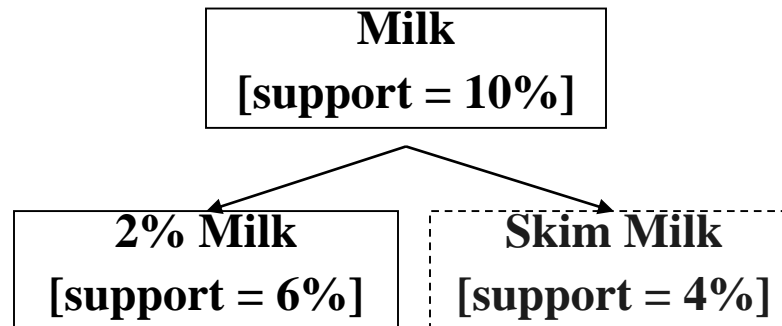- Pattern Exploration and Application

- Summary

Research on Pattern Mining: A Road Map

Research on Pattern Mining

- **Kinds of patterns and rules**
  - **Basic Patterns**
    - frequent pattern
    - association rule
    - closed/max pattern
    - generator
  - **Multilevel & Multidimensional Patterns**
    - multilevel (uniform, varied, or itemset -based support)
    - multidimensional pattern (incl . high-dimensional pattern)
    - continuous data (discretization -based, or statistical)
  - **Extended Patterns**
    - approximate pattern
    - uncertain pattern
    - compressed pattern
    - rare pattern/negative pattern
    - high-dimensional and colossal patterns

- **Mining Methods**
  - **Basic Mining Methods**
    - candidate generation (Apriori , partitioning, sampling, ...)
    - Pattern growth (FPgrowth , HMine , FPMax, Closet+, ...)
    - vertical format (EClat , CHARM, ...)
  - **Mining Interesting Patterns**
    - interestingness (subjective vs. objective)
    - constraint-based mining
    - correlation rules
    - exception rules
  - **Distributed, parallel & incremental**
    - distributed/parallel mining
    - incremental mining
    - stream pattern

- **Extensions & Application**
  - **Extended Data Types**
    - sequential ad time-series patterns
    - structural (e.g., tree, lattice, graph) patterns
    - spatial (e.g., co-location) pattern
    - temporal (evolutionary, periodic)
    - image, video and multimedia patterns
    - network patterns
  - **Applications**
    - pattern-based classification
    - pattern-based clustering
    - pattern-based semantic annotation
    - collaborative filtering
    - privacy-preserving

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- Pattern Mining in Multi-Level, Multi-Dimensional Space

  - Mining Multi-Level Association

  - Mining Multi-Dimensional Association

  - Mining Quantitative Association Rules

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Mining Multiple-Level Association Rules

- Items often form hierarchies
- Flexible support settings
  - Items at the lower level are expected to have lower support
- Exploration of *shared* multi-level mining (Agrawal & Srikant@VLB'95, Han & Fu@VLDB'95)

uniform support   **reduced support**

Level 1
min_sup = 5%

Level 2
min_sup = 5%

Milk
[support = 10%]

2% Milk
[support = 6%]

Skim Milk
[support = 4%]

Level 1
min_sup = 5%

Level 2
min_sup = 3%

Multi-level Association: Flexible Support and Redundancy filtering

- Flexible min-support thresholds: Some items are more valuable but less frequent
  - Use non-uniform, group-based min-support
  - E.g., {diamond, watch, camera}: 0.05%; {bread, milk}: 5%; …
- Redundancy Filtering: Some rules may be redundant due to "ancestor" relationships between items
  - milk $\Rightarrow$ wheat bread [support = 8%, confidence = 70%]
  - 2% milk $\Rightarrow$ wheat bread [support = 2%, confidence = 72%]

  The first rule is an ancestor of the second rule
- A rule is *redundant* if its support is close to the "expected" value, based on the rule's ancestor

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Mining Multi-Dimensional Association

- Single-dimensional rules:

    buys(X, "milk") $\Rightarrow$ buys(X, "bread")

- Multi-dimensional rules: $\geq$ 2 dimensions or predicates

    - Inter-dimension assoc. rules (*no repeated predicates*)

        age(X,"19-25") $\wedge$ occupation(X,"student") $\Rightarrow$ buys(X, "coke")

    - hybrid-dimension assoc. rules (*repeated predicates*)

        age(X,"19-25") $\wedge$ buys(X, "popcorn") $\Rightarrow$ buys(X, "coke")

- Categorical Attributes: finite number of possible values, no ordering among values—data cube approach

- Quantitative Attributes: Numeric, implicit ordering among values—discretization, clustering, and gradient approaches

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Mining Quantitative Associations

Techniques can be categorized by how numerical attributes, such as age or salary are treated

1. Static discretization based on predefined concept hierarchies (data cube methods)

2. Dynamic discretization based on data distribution (quantitative rules, e.g., Agrawal & Srikant@SIGMOD96)

3. Clustering: Distance-based association (e.g., Yang & Miller@SIGMOD97)

   ■ One dimensional clustering then association

4. Deviation: (such as Aumann and Lindell@KDD99)

   Sex = female => Wage: mean=$7/hr (overall mean = $9)

Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy.

- Numeric values are replaced by ranges

- In relational database, finding all frequent k-predicate sets will require k or $k+1$ table scans

- Data cube is well suited for mining

- The cells of an n-dimensional cuboid correspond to the predicate sets

- Mining from data cubes can be much faster

()

(age)   (income)   (buys)

(age, income)   (age,buys)   (income,buys)

(age,income,buys)
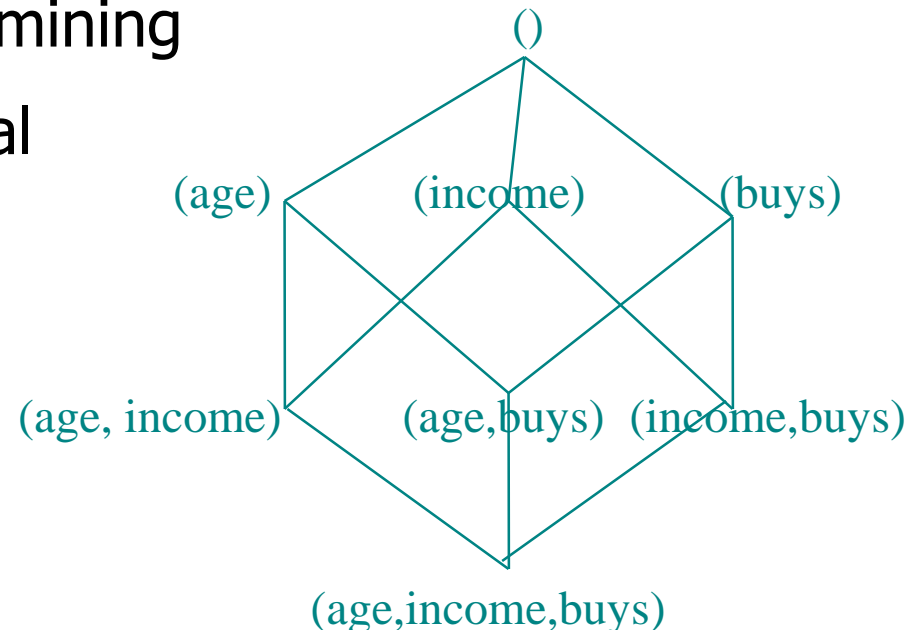
Quantitative Association Rules Based on Statistical Inference Theory [Aumann and Lindell@DMKD'03]

- Finding extraordinary and therefore interesting phenomena, e.g.,

    (Sex = female) => Wage: mean=$7/hr (overall mean = $9)

  - LHS: a subset of the population
  - RHS: an extraordinary behavior of this subset

- The rule is accepted only if a statistical test (e.g., Z-test) confirms the inference with high confidence

- Subrule: highlights the extraordinary behavior of a subset of the pop. of the super rule

  - E.g., (Sex = female) ^ (South = yes) => mean wage = $6.3/hr

- Two forms of rules

  - Categorical => quantitative rules, or Quantitative => quantitative rules
  - E.g., Education in [14-18] (yrs) => mean wage = $11.64/hr

- Open problem: Efficient methods for LHS containing two or more quantitative attributes

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Negative and Rare Patterns

- Rare patterns: Very low support but interesting
    - E.g., buying Rolex watches
    - Mining: Setting individual-based or special group-based support threshold for valuable items
- Negative patterns
    - Since it is unlikely that one buys Ford Expedition (an SUV car) and Toyota Prius (a hybrid car) together, Ford Expedition and Toyota Prius are likely negatively correlated patterns
- Negatively correlated patterns that are infrequent tend to be more interesting than those that are frequent

Defining Negative Correlated Patterns (I)

- Definition 1 (support-based)

  - If itemsets X and Y are both frequent but rarely occur together, i.e.,

    sup(X U Y) < sup (X) * sup(Y)

  - Then X and Y are negatively correlated

- Problem: A store sold two needle 100 packages A and B, only one transaction containing both A and B.

  - When there are in total 200 transactions, we have

    s(A U B) = 0.005, s(A) * s(B) = 0.25, s(A U B) < s(A) * s(B)

  - When there are 10^5 transactions, we have

    s(A U B) = $1/10^5$, s(A) * s(B) = $1/10^3 * 1/10^3$, s(A U B) > s(A) * s(B)

  - Where is the problem? —Null transactions, i.e., the support-based definition is not null-invariant!

Defining Negative Correlated Patterns (II)

- Definition 2 (negative itemset-based)

  - X is a *negative itemset* if (1) X = Ā ∪ B, where B is a set of positive items, and Ā is a set of negative items, |Ā| ≥ 1, and (2) s(X) ≥ μ

  - Itemsets X is negatively correlated, if

$$s(X) < \prod_{i=1}^{k} s(x_i), where\ x_i \in X,\ and\ s(x_i)\ is\ the\ support\ of\ x_i$$

- This definition suffers a similar null-invariant problem

- Definition 3 (Kulzynski measure-based) If itemsets X and Y are frequent, but (P(X|Y) + P(Y|X))/2 < ϵ, where ϵ is a negative pattern threshold, then X and Y are negatively correlated.

- Ex. For the same needle package problem, when no matter there are 200 or 10^5 transactions, if ϵ = 0.01, we have

  (P(A|B) + P(B|A))/2 = (0.01 + 0.01)/2 < ϵ

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Constraint-based (Query-Directed) Mining

- Finding all the patterns in a database autonomously? — unrealistic!
    - The patterns could be too many but not focused!
- Data mining should be an interactive process
    - User directs what to be mined using a data mining query language (or a graphical user interface)
- Constraint-based mining
    - User flexibility: provides constraints on what to be mined
    - Optimization: explores such constraints for efficient mining — constraint-based mining: constraint-pushing, similar to push selection first in DB query processing
    - Note: still find all the answers satisfying constraints, not finding some answers in "heuristic search"

Constraints in Data Mining

- Knowledge type constraint:
  - classification, association, etc.
- Data constraint — using SQL-like queries
  - find product pairs sold together in stores in Chicago this year
- Dimension/level constraint
  - in relevance to region, price, brand, customer category
- Rule (or pattern) constraint
  - small sales (price < $10) triggers big sales (sum > $200)
- Interestingness constraint
  - strong rules: min_support $\geq$ 3%, min_confidence $\geq$ 60%

Meta-Rule Guided Mining

- Meta-rule can be in the rule form with partially instantiated predicates and constants

$$P_1(X, Y) \wedge P_2(X, W) => buys(X, \text{"iPad"})$$

- The resulting rule derived can be

$$age(X, \text{"15-25"}) \wedge profession(X, \text{"student"}) => buys(X, \text{"iPad"})$$

- In general, it can be in the form of

$$P_1 \wedge P_2 \wedge ... \wedge P_l => Q_1 \wedge Q_2 \wedge ... \wedge Q_r$$

- Method to find meta-rules

  - Find frequent (l+r) predicates (based on min-support threshold)

  - Push constants deeply when possible into the mining process (see the remaining discussions on constraint-push techniques)

  - Use confidence, correlation, and other measures when possible

Constraint-Based Frequent Pattern Mining

- Pattern space pruning constraints

    - Anti-monotonic: If constraint c is violated, its further mining can be terminated

    - Monotonic: If c is satisfied, no need to check c again

    - Succinct: c must be satisfied, so one can start with the data sets satisfying c

    - Convertible: c is not monotonic nor anti-monotonic, but it can be converted into it if items in the transaction can be properly ordered

- Data space pruning constraint

    - Data succinct: Data space can be pruned at the initial pattern mining process

    - Data anti-monotonic: If a transaction t does not satisfy c, t can be pruned from its further mining

Pattern Space Pruning with Anti-Monotonicity Constraints

- A constraint C is *anti-monotone* if the super pattern satisfies C, all of its sub-patterns do so too

- In other words, *anti-monotonicity:* If an itemset S **violates** the constraint, so does any of its superset

- Ex. 1. $sum(S.price) \leq v$ is anti-monotone

- Ex. 2. range(S.profit) $\leq$ 15 is anti-monotone
  - Itemset *ab* violates C
  - So does every superset of *ab*

- Ex. 3. $sum(S.Price) \geq v$ is not anti-monotone

- Ex. 4. *support count* is anti-monotone: core property used in Apriori

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

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Pattern Space Pruning with Monotonicity Constraints

- A constraint C is *monotone* if the pattern satisfies C, we do not need to check C in subsequent mining

- Alternatively, monotonicity: *If an itemset S* **satisfies** *the constraint, so does any of its superset*

- Ex. 1. *sum(S.Price)* $\geq$ *v* is monotone

- Ex. 2. *min(S.Price)* $\leq$ *v* is monotone

- Ex. 3. C: range(S.profit) $\geq$ 15

  - Itemset *ab* satisfies C

  - So does every superset of *ab*

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Data Space Pruning with Data Anti-monotonicity

- A constraint c is *data anti-monotone* if for a pattern p cannot satisfy a transaction t under c, p's superset cannot satisfy t under c either

- The key for data anti-monotone is *recursive data reduction*

- Ex. 1. *sum(S.Price)* $\geq$ *v* is data anti-monotone

- Ex. 2. *min(S.Price)* $\leq$ *v* is data anti-monotone

- Ex. 3. C: *range(S.profit)* $\geq$ *25* is data anti-monotone

  - Itemset {b, c}'s projected DB:
    - T10': {d, f, h}, T20': {d, f, g, h}, T30': {d, f, g}

  - since C cannot satisfy T10', T10' can be pruned

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	-5

Pattern Space Pruning with Succinctness

- Succinctness:

  - Given A_1, the set of items satisfying a succinctness constraint C, then any set S satisfying C is based on A_1, i.e., S contains a subset belonging to A_1

  - Idea: Without looking at the transaction database, whether an itemset S satisfies constraint C can be determined based on the selection of items

  - $min(S.Price) \leq v$ is succinct

  - $sum(S.Price) \geq v$ is not succinct

- Optimization: If C is succinct, C is pre-counting pushable

Naïve Algorithm: Apriori + Constraint

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D →

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

→

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
~~{5}~~	~~3~~

L_2

itemset	sup
{1 3}	2
~~{2 3}~~	~~2~~
~~{2 5}~~	~~3~~
~~{3 5}~~	~~2~~

←

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

← Scan D ←

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

C_3

itemset
{2 3 5}

Scan D →

L_3

itemset	sup
~~{2 3 5}~~	~~2~~

Constraint:

Sum{S.price} < 5

Constrained Apriori : Push a Succinct Constraint Deep

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D →

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

not immediately to be used

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

← Scan D

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D →

L_3

itemset	sup
{2 3 5}	2

Constraint:

min{S.price } <= 1

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Constrained FP-Growth: Push a Succinct Constraint Deep

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Remove infrequent length 1

TID	Items
100	1 3
200	2 3 5
300	1 2 3 5
400	2 5

FP-Tree
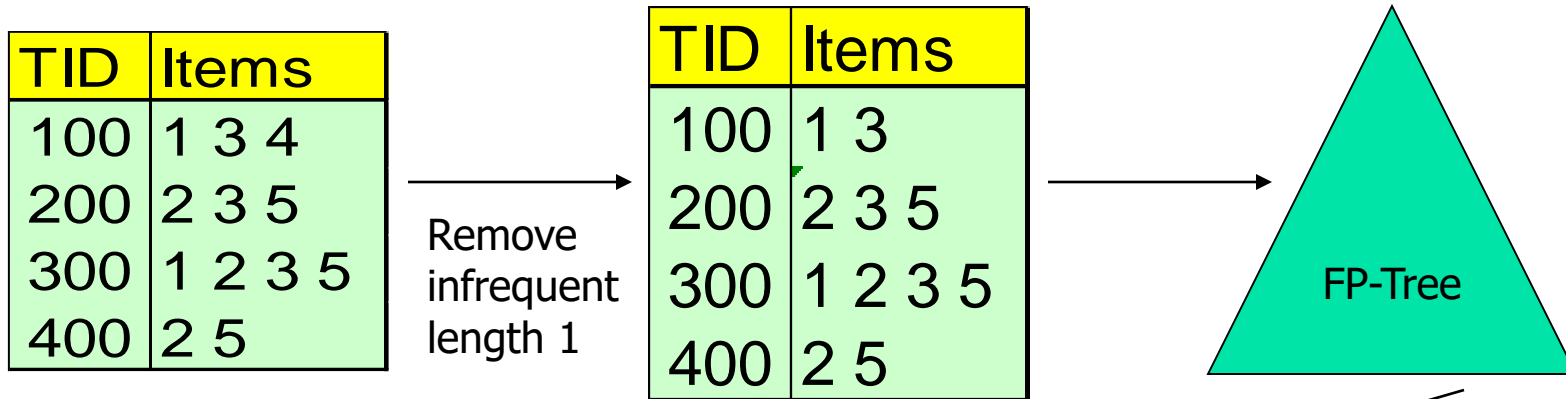
1-Projected DB

TID	Items
100	3 4
300	2 3 5

No Need to project on 2, 3, or 5

Constraint:

$$\min\{S.price\} <= 1$$

Constrained FP-Growth: Push a Data Anti-monotonic Constraint Deep

Remove from data

TID	Items
100	1 3 4
~~200~~	~~2 3 5~~
300	1 2 3 5
~~400~~	~~2 5~~

TID	Items
100	1 3
300	1 3

FP-Tree
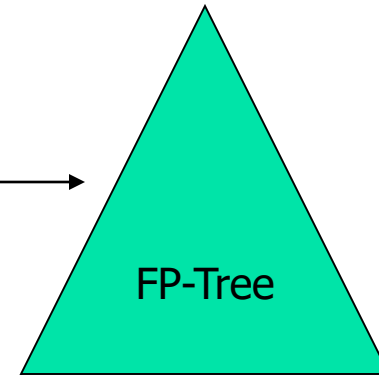
Single branch, we are done

Constraint:

min{S.price } <= 1

Constrained FP-Growth: Push a Data Anti-monotonic Constraint Deep

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

FP-Tree

Recursive Data Pruning

B-Projected DB

TID	Transaction
10	a, c, d, f, h
20	c, d, f, g, h
30	c, d, f, g

B FP-Tree

Single branch:

bcdfg: 2
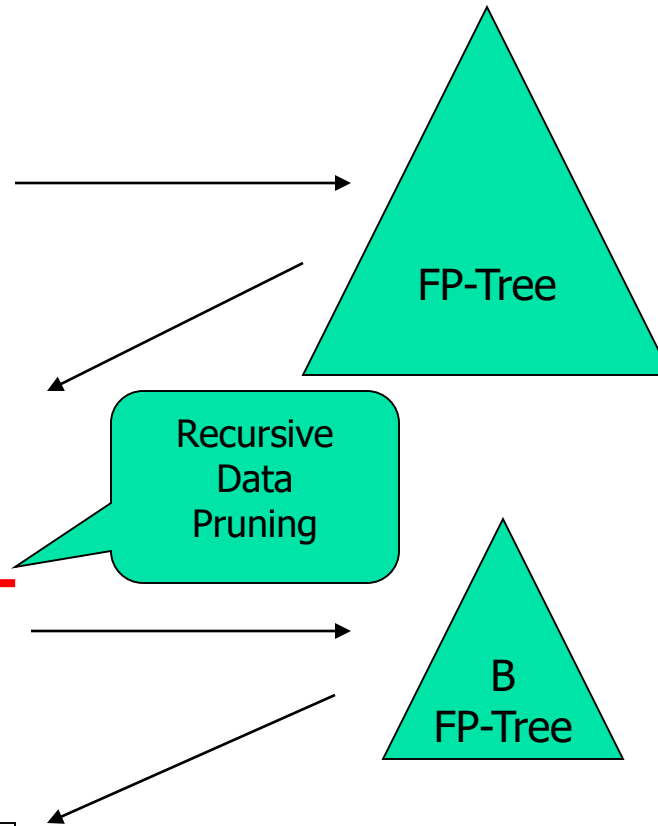
Item	Profit
a	40
b	0
c	-20
d	-15
e	-30
f	-10
g	20
h	-5

Constraint:
range{S.price } > 25
min_sup >= 2

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Convertible Constraints: Ordering Data in Transactions

TDB (min_sup=2)

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

- Convert tough constraints into anti-monotone or monotone by properly ordering items

- Examine C: avg(*S*.profit) ≥ 25

  - Order items in value-descending order

    - *<a, f, g, d, b, h, c, e>*

  - If an itemset *afb* violates C

    - So does *afbh, afb**

    - It becomes anti-monotone!

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Strongly Convertible Constraints

- avg(X) ≥ 25 is convertible anti-monotone w.r.t. item value descending order R: <*a, f, g, d, b, h, c, e*>
  - If an itemset *af* violates a constraint C, so does every itemset with *af* as prefix, such as *afd*
- avg(X) ≥ 25 is convertible monotone w.r.t. item value ascending order R^{-1}: <*e, c, h, b, d, g, f, a*>
  - If an itemset *d* satisfies a constraint *C*, so does itemsets *df* and *dfa*, which having *d* as a prefix
- Thus, avg(X) ≥ 25 is strongly convertible

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Can Apriori Handle Convertible Constraints?

- A convertible, not monotone nor anti-monotone nor succinct constraint cannot be pushed deep into the an Apriori mining algorithm

  - Within the level wise framework, no direct pruning based on the constraint can be made

  - Itemset df violates constraint C: avg(X) >= 25

  - Since adf satisfies C, Apriori needs df to assemble adf, df cannot be pruned

- But it can be pushed into frequent-pattern growth framework!

Item	Value
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Pattern Space Pruning w. Convertible Constraints

Item	Value
a	40
f	30
g	20
d	10
b	0
h	-10
c	-20
e	-30

- C: avg(X) >= 25, min_sup=2
- List items in every transaction in value descending order R: <a, f, g, d, b, h, c, e>
  - C is convertible anti-monotone w.r.t. R
- Scan TDB once
  - remove infrequent items
    - Item h is dropped
  - Itemsets a and f are good, …
- Projection-based mining
  - Imposing an appropriate order on item projection
  - Many tough constraints can be converted into (anti)-monotone

TDB (min_sup=2)

TID	Transaction
10	a, f, d, b, c
20	f, g, d, b, c
30	a, f, d, c, e
40	f, g, h, c, e

Handling Multiple Constraints

- Different constraints may require different or even conflicting item-ordering

- If there exists an order R s.t. both C_1 and C_2 are convertible w.r.t. R, then there is no conflict between the two convertible constraints

- If there exists conflict on order of items

  - Try to satisfy one constraint first

  - Then using the order for the other constraint to mine frequent itemsets in the corresponding projected database

What Constraints Are Convertible?

Constraint	Convertible anti-monotone	Convertible monotone	Strongly convertible
avg(S) ≤ , ≥ v	Yes	Yes	Yes
median(S) ≤ , ≥ v	Yes	Yes	Yes
sum(S) ≤ v (items could be of any value, v ≥ 0)	Yes	No	No
sum(S) ≤ v (items could be of any value, v ≤ 0)	No	Yes	No
sum(S) ≥ v (items could be of any value, v ≥ 0)	No	Yes	No
sum(S) ≥ v (items could be of any value, v ≤ 0)	Yes	No	No
……			

Constraint-Based Mining — A General Picture

Constraint	Anti-monotone	Monotone	Succinct
$v \in S$	no	yes	yes
$S \supseteq V$	no	yes	yes
$S \subseteq V$	yes	no	yes
$\min(S) \leq v$	no	yes	yes
$\min(S) \geq v$	yes	no	yes
$\max(S) \leq v$	yes	no	yes
$\max(S) \geq v$	no	yes	yes
$\text{count}(S) \leq v$	yes	no	weakly
$\text{count}(S) \geq v$	no	yes	weakly
$\text{sum}(S) \leq v$ ($a \in S, a \geq 0$)	yes	no	no
$\text{sum}(S) \geq v$ ($a \in S, a \geq 0$)	no	yes	no
$\text{range}(S) \leq v$	yes	no	no
$\text{range}(S) \geq v$	no	yes	no
$\text{avg}(S) \ \theta \ v, \ \theta \in \{ =, \leq, \geq \}$	convertible	convertible	no
$\text{support}(S) \geq \xi$	yes	no	no
$\text{support}(S) \leq \xi$	no	yes	no

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- Summary

Mining Colossal Frequent Patterns

- F. Zhu, X. Yan, J. Han, P. S. Yu, and H. Cheng, "Mining Colossal Frequent Patterns by Core Pattern Fusion", ICDE'07.

- We have many algorithms, but can we mine large (i.e., colossal) patterns? — such as just size around 50 to 100? Unfortunately, not!

- Why not? — the curse of "downward closure" of frequent patterns

  - The "downward closure" property

    - Any sub-pattern of a frequent pattern is frequent.

  - Example. If $(a_1, a_2, \ldots, a_{100})$ is frequent, then $a_1, a_2, \ldots, a_{100}, (a_1, a_2), (a_1, a_3), \ldots, (a_1, a_{100}), (a_1, a_2, a_3), \ldots$ are all frequent! There are about 2^{100} such frequent itemsets!

  - No matter using breadth-first search (e.g., Apriori) or depth-first search (FPgrowth), we have to examine so many patterns

- Thus the downward closure property leads to explosion!

Colossal Patterns: A Motivating Example

Let's make a set of 40 transactions

   T1 = 1 2 3 4 ..... 39 40

   T2 = 1 2 3 4 ..... 39 40

   ⋮   .

   ⋮   .

   ⋮   .

   ⋮   .

   T40=1 2 3 4 ..... 39 40

Then delete the items on the diagonal

   T_1 = 2 3 4 ..... 39 40

   T_2 = 1 3 4 ..... 39 40

   ⋮   .

   ⋮   .

   ⋮   .

   ⋮   .

   T_{40}=1 2 3 4 39

Closed/maximal patterns may partially alleviate the problem but not really solve it: We often need to mine scattered large patterns!

Let the minimum support threshold σ= 20

There are $\binom{40}{20}$ frequent patterns of size 20

Each is closed and maximal

patterns = $\binom{n}{n/2} \approx \sqrt{2/\pi} \dfrac{2^n}{\sqrt{n}}$

The size of the answer set is exponential to n

Colossal Pattern Set: Small but Interesting

- It is often the case that only a small number of patterns are colossal, i.e., of large size

- Colossal patterns are usually attached with greater importance than those of small pattern sizes
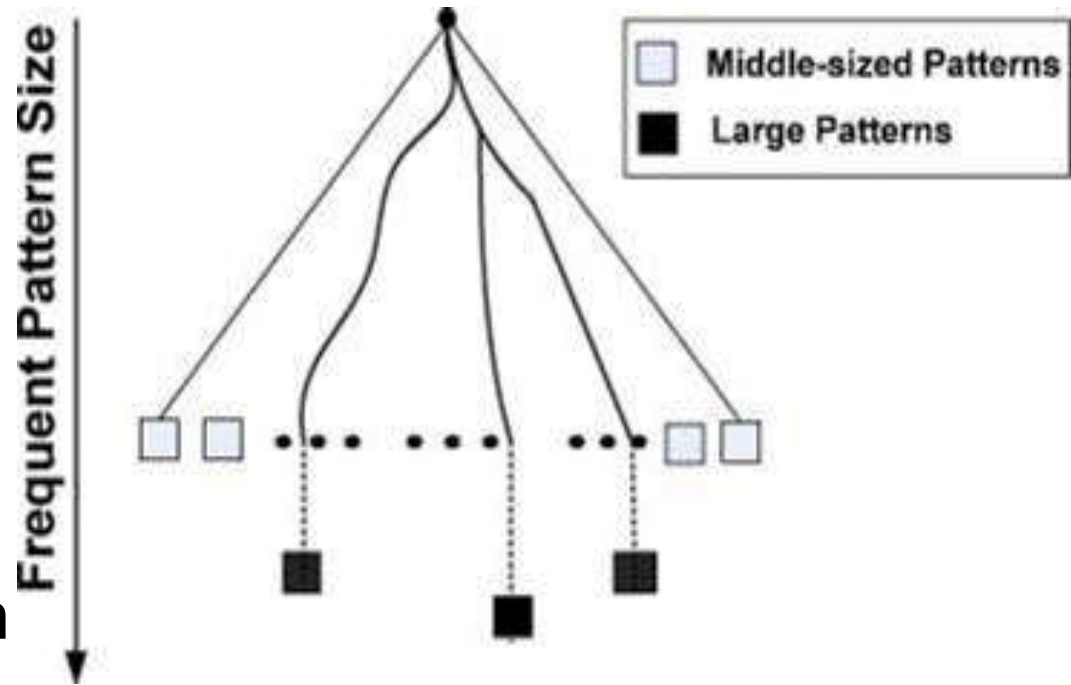
Mining Colossal Patterns: Motivation and Philosophy

- Motivation: Many real-world tasks need mining colossal patterns
  - Micro-array analysis in bioinformatics (when support is low)
  - Biological sequence patterns
  - Biological/sociological/information graph pattern mining
- *No hope for completeness*
  - If the mining of mid-sized patterns is explosive in size, there is no hope to find colossal patterns efficiently by insisting "complete set" mining philosophy
- *Jumping out of the swamp of the mid-sized results*
  - What we may develop is a philosophy that may jump out of the swamp of mid-sized results that are explosive in size and jump to reach colossal patterns
- *Striving for mining almost complete colossal patterns*
  - The key is to develop a mechanism that may quickly reach colossal patterns and discover most of them

Alas, A Show of Colossal Pattern Mining!

T_1 = 2 3 4 ..... 39 40

T_2 = 1 3 4 ..... 39 40

: .

: .

: .

: .

T_{40}=1 2 3 4 39

T_{41}= 41 42 43 79

T_{42}= 41 42 43 79

: .

: .

T_{60}= 41 42 43 ... 79

Let the min-support threshold σ= 20

Then there are $\binom{40}{20}$ closed/maximal frequent patterns of size 20

However, there is only one with size greater than 20, (*i.e.,* colossal):

α= {41,42,…,79} of size 39

The existing fastest mining algorithms (*e.g.,* FPClose, LCM) fail to complete running

Our algorithm outputs this colossal pattern in seconds

Methodology of Pattern-Fusion Strategy

- Pattern-Fusion traverses the tree in a bounded-breadth way

  - Always pushes down a frontier of a bounded-size candidate pool

  - Only a fixed number of patterns in the current candidate pool will be used as the starting nodes to go down in the pattern tree — thus avoids the exponential search space

- Pattern-Fusion identifies "shortcuts" whenever possible

  - Pattern growth is not performed by single-item addition but by leaps and bounded: agglomeration of multiple patterns in the pool

  - These shortcuts will direct the search down the tree much more rapidly towards the colossal patterns

Observation: Colossal Patterns and Core Patterns

A colossal pattern α

Transaction Database D
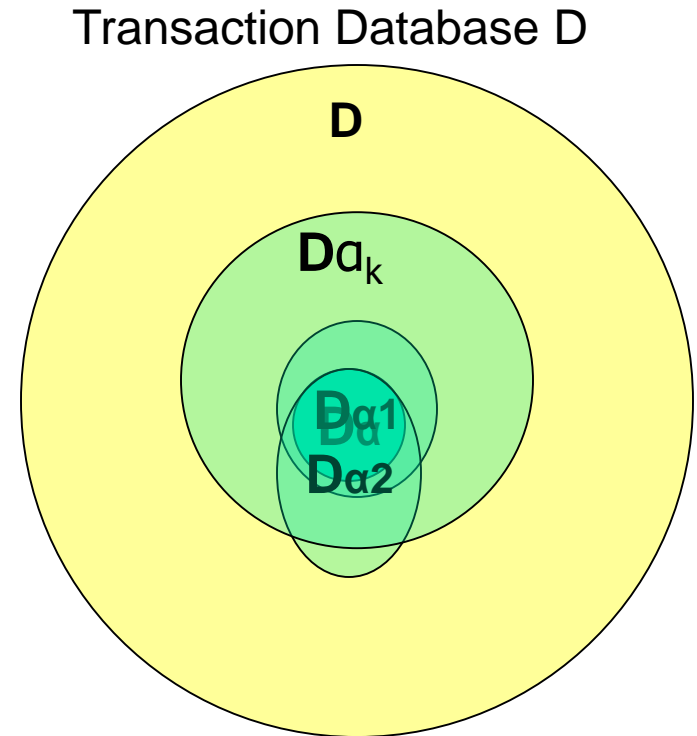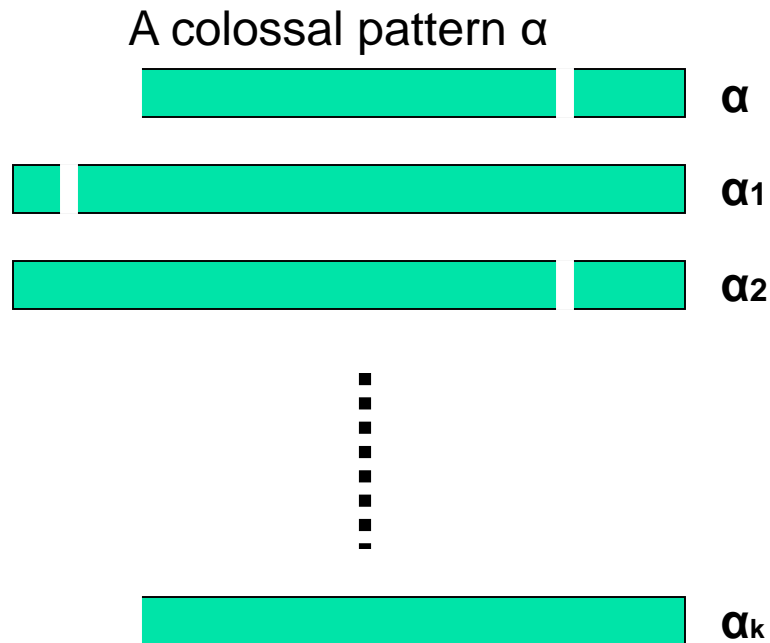
Subpatterns α_1 to α_k cluster tightly around the colossal pattern α by sharing a similar support. We call such subpatterns *core patterns* of α

Robustness of Colossal Patterns

- Core Patterns

  Intuitively, for a frequent pattern α, a subpattern β is a τ-core pattern of α if β shares a similar support set with α, i.e.,

  $$\frac{|D_\alpha|}{|D_\beta|} \geq \tau \qquad 0 < \tau \leq 1$$

  where τ is called the core ratio

- Robustness of Colossal Patterns

  A colossal pattern is robust in the sense that it tends to have much more core patterns than small patterns

Example: Core Patterns

- A colossal pattern has far more core patterns than a small-sized pattern
- A colossal pattern has far more core descendants of a smaller size c
- A random draw from a complete set of pattern of size c would more likely to pick a core descendant of a colossal pattern
- A colossal pattern can be generated by merging a set of core patterns

Transaction (# of Ts)	Core Patterns ($\tau = 0.5$)
(abe) (100)	(abe), (ab), (be), (ae), (e)
(bcf) (100)	(bcf), (bc), (bf)
(acf) (100)	(acf), (ac), (af)
(abcef) (100)	(ab), (ac), (af), (ae), (bc), (bf), (be) (ce), (fe), (e), (abc), (abf), (abe), (ace), (acf), (afe), (bcf), (bce), (bfe), (cfe), (abcf), (abce), (bcfe), (acfe), (abfe), (abcef)

Colossal Patterns Correspond to Dense Balls

- Due to their robustness, colossal patterns correspond to dense balls

  - $\Omega(2^d)$ in population

- A random draw in the pattern space will hit somewhere in the ball with high probability
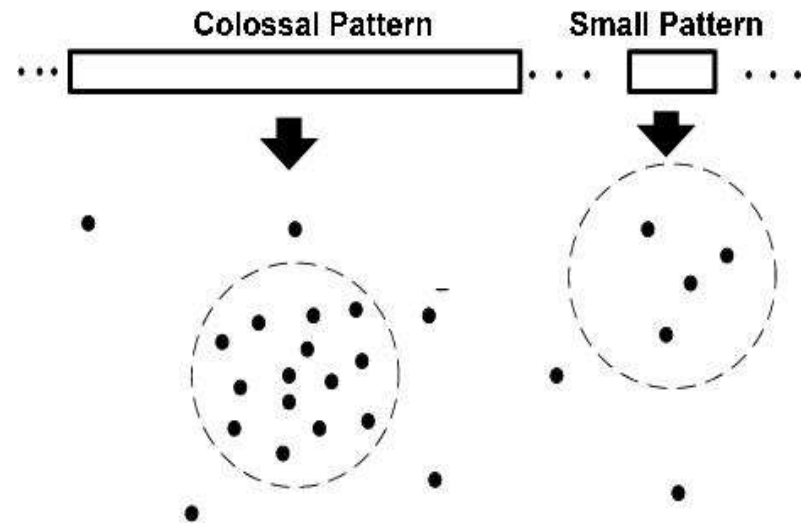
Idea of Pattern-Fusion Algorithm

- Generate a complete set of frequent patterns up to a small size

- Randomly pick a pattern β, and β has a high probability to be a core-descendant of some colossal pattern α

- Identify all α's descendants in this complete set, and merge all of them — This would generate a much larger core-descendant of α

- In the same fashion, we select K patterns. This set of larger core-descendants will be the candidate pool for the next iteration

Pattern-Fusion: The Algorithm

- Initialization (Initial pool): Use an existing algorithm to mine all frequent patterns up to a small size, e.g., 3
- Iteration (Iterative Pattern Fusion):
    - At each iteration, k seed patterns are randomly picked from the current pattern pool
    - For each seed pattern thus picked, we find all the patterns within a bounding ball centered at the seed pattern
    - All these patterns found are fused together to generate a set of super-patterns. All the super-patterns thus generated form a new pool for the next iteration
- Termination: when the current pool contains no more than K patterns at the beginning of an iteration

Why Is Pattern-Fusion Efficient?

- A bounded-breadth pattern tree traversal

    - It avoids explosion in mining mid-sized ones
    - Randomness comes to help to stay on the right path
- Ability to identify "short-cuts" and take "leaps"

    - fuse small patterns together in one step to generate new patterns of significant sizes
    - Efficiency
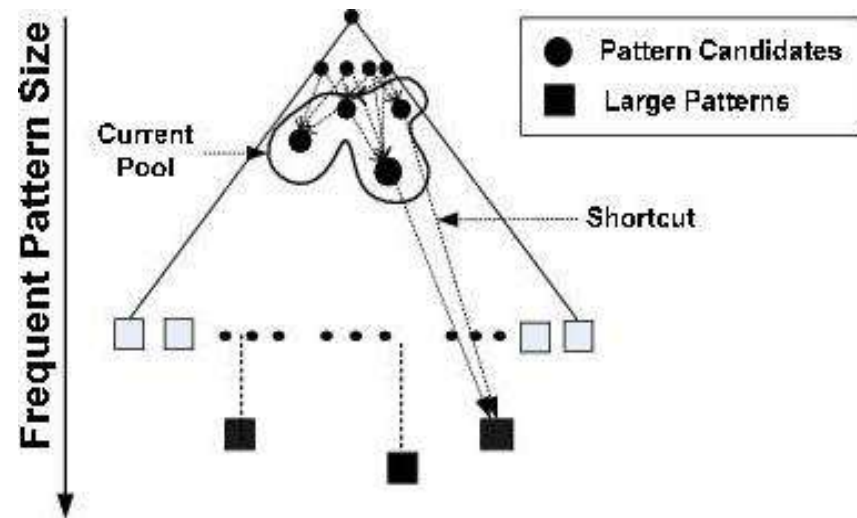
Pattern-Fusion Leads to Good Approximation

- Gearing toward colossal patterns
  - The larger the pattern, the greater the chance it will be generated
- Catching outliers
  - The more distinct the pattern, the greater the chance it will be generated

Experimental Setting

- Synthetic data set

  - $Diag_n$ an n x (n-1) table where i^{th} row has integers from 1 to n except i. Each row is taken as an itemset. min_support is n/2.

- Real data set

  - Replace: A program trace data set collected from the "replace" program, widely used in software engineering research

  - ALL: A popular gene expression data set, a clinical data on ALL-AML leukemia (www.broad.mit.edu/tools/data.html).

    - Each item is a column, representing the activitiy level of gene/protein in the same

    - Frequent pattern would reveal important correlation between gene expression patterns and disease outcomes

Experiment Results on Diag$_n$

- LCM run time increases exponentially with pattern size n

- Pattern-Fusion finishes efficiently

- The approximation error of Pattern-Fusion (with min-sup 20) in comparison with the complete set) is rather close to uniform sampling (which randomly picks K patterns from the complete answer set)
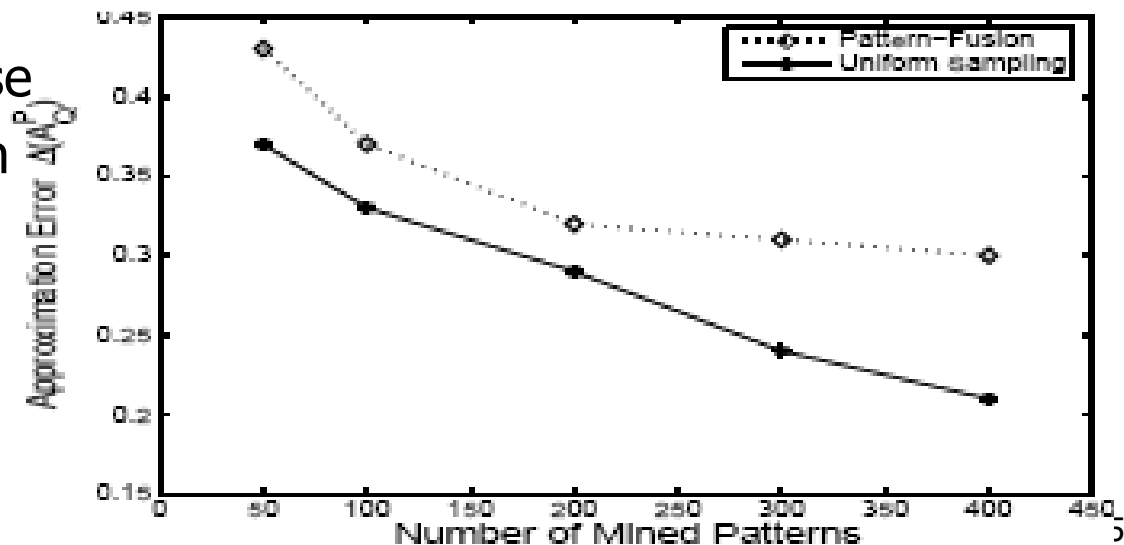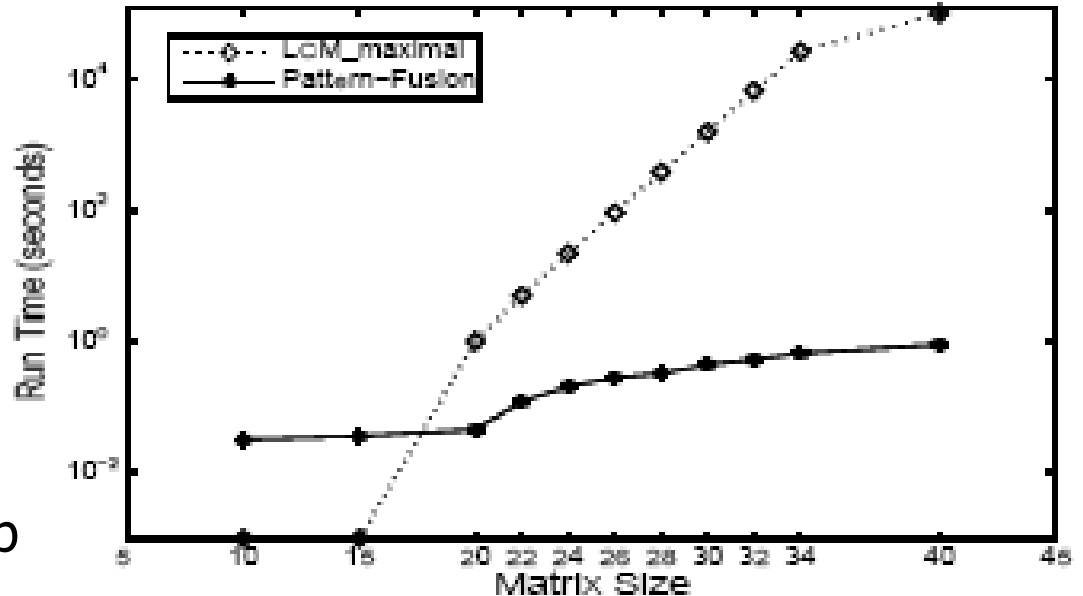
Experimental Results on ALL

- ALL: A popular gene expression data set with 38 transactions, each with 866 columns
  - There are 1736 items in total
  - The table shows a high frequency threshold of 30

Pattern Size	110	107	102	91	86	84	83
The complete set	1	1	1	1	1	2	6
Pattern-Fusion	1	1	1	1	1	1	4
Pattern Size	82	77	76	75	74	73	71
The complete set	1	2	1	1	1	2	1
Pattern-Fusion	0	2	0	1	1	1	1
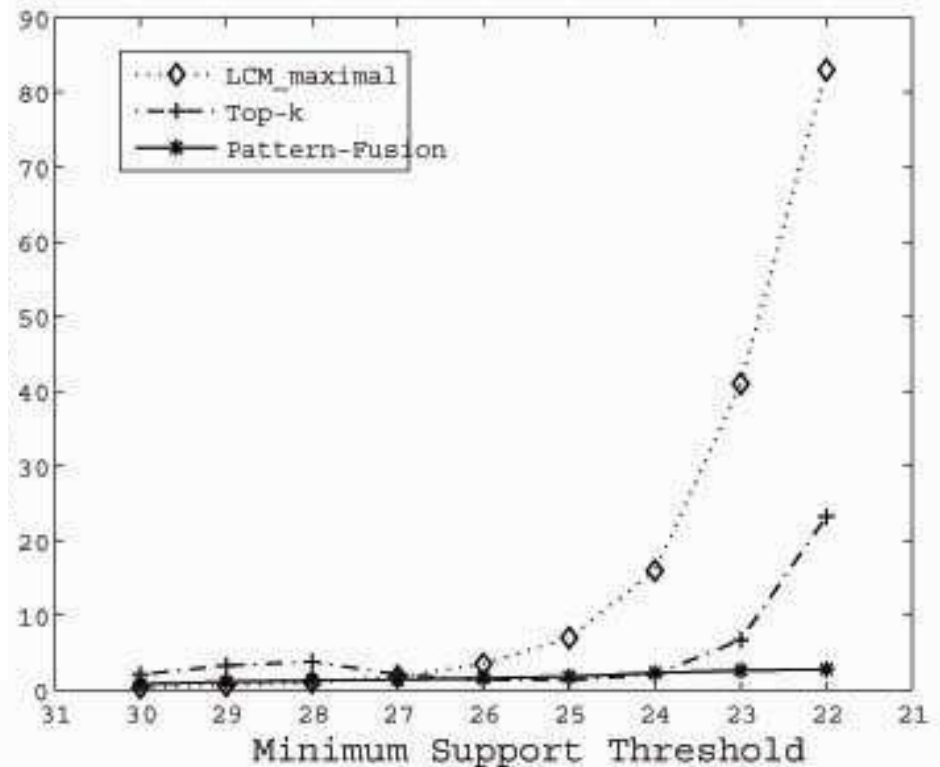
Experimental Results on REPLACE

- REPLACE
  - A program trace data set, recording 4395 calls and transitions
  - The data set contains 4395 transactions with 57 items in total
  - With support threshold of 0.03, the largest patterns are of size 44
  - They are all discovered by Pattern-Fusion with different settings of K and τ, when started with an initial pool of 20948 patterns of size <=3

Experimental Results on REPLACE

- Approximation error when compared with the complete mining result

- Example. Out of the total 98 patterns of size >=42, when K=100, Pattern-Fusion returns 80 of them

- A good approximation to the colossal patterns in the sense that any pattern in the complete set is on average at most 0.17 items away from one of these 80 patterns
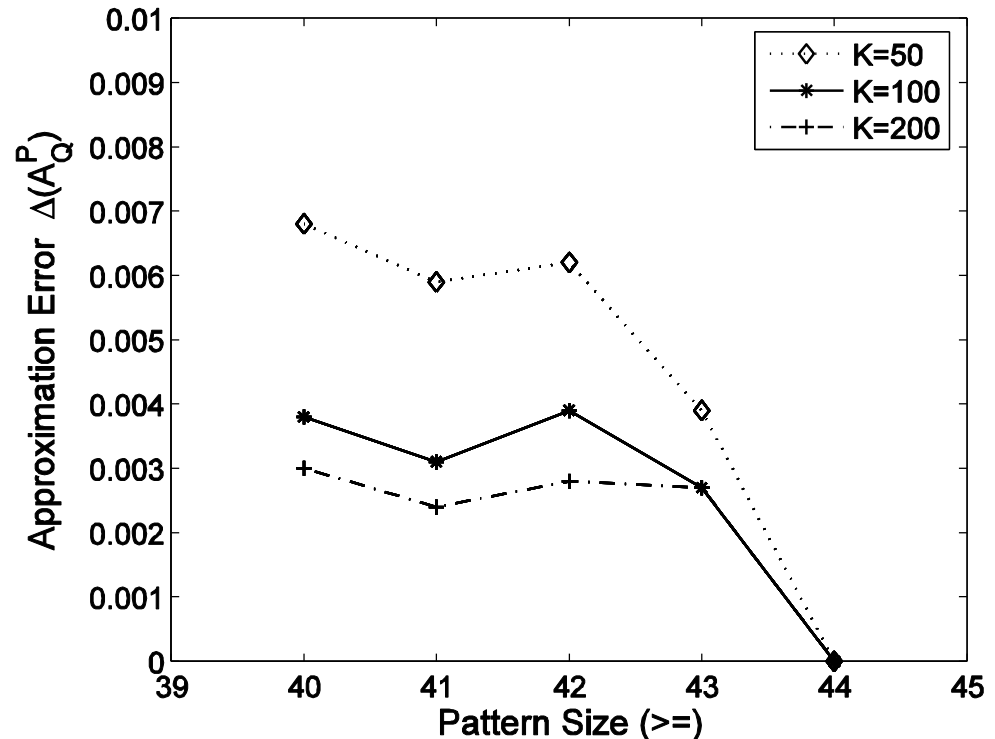
Chapter 7 : Advanced Frequent Pattern Mining

- **Pattern Mining: A Road Map**

- **Pattern Mining in Multi-Level, Multi-Dimensional Space**

- **Constraint-Based Frequent Pattern Mining**

- **Mining High-Dimensional Data and Colossal Patterns**

- **Mining Compressed or Approximate Patterns**

- **Pattern Exploration and Application**

- **Summary**

Mining Compressed Patterns: δ-clustering

- Why compressed patterns?
  - too many, but less meaningful
- Pattern distance measure

$$D(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|}$$

ID	Item-Sets	Support
P1	{38,16,18,12}	205227
P2	{38,16,18,12,17}	205211
P3	{39,38,16,18,12,17}	101758
P4	{39,16,18,12,17}	161563
P5	{39,16,18,12}	161576

- δ-clustering: For each pattern P, find all patterns which can be expressed by P and their distance to P are within δ (δ-cover)
- All patterns in the cluster can be represented by P
- Xin et al., "Mining Compressed Frequent-Pattern Sets", VLDB'05

- Closed frequent pattern
  - Report P1, P2, P3, P4, P5
  - Emphasize too much on support
  - no compression
- Max-pattern, P3: info loss
- A desirable output: P2, P3, P4

Redundancy-Award Top-k Patterns

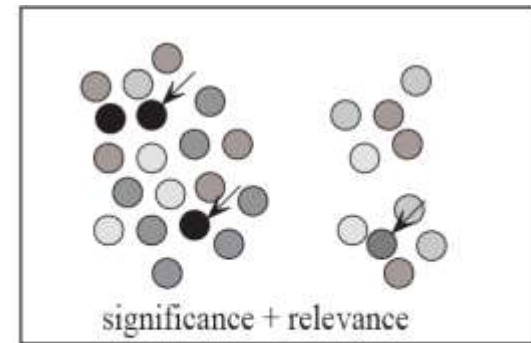- Why redundancy-aware top-k patterns?

- Desired patterns: high significance & low redundancy

- Propose the MMS (Maximal Marginal Significance) for measuring the combined significance of a pattern set
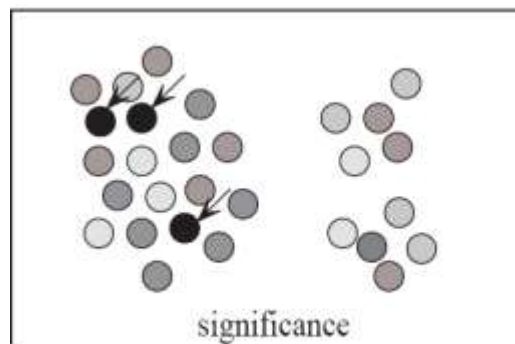
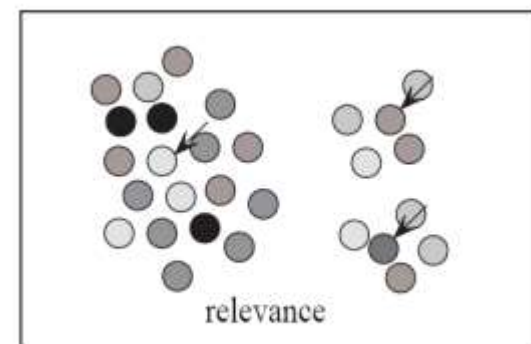- Xin et al., Extracting Redundancy-Aware Top-K Patterns, KDD'06
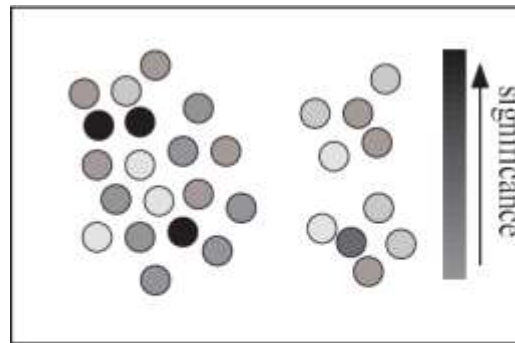
(a) a set of patterns

(b) redundancy-aware top-k

(c) traditional top-k

(d) summarization

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How to Understand and Interpret Patterns?

diaper   beer

female sterile (2) tekele

- Do they all make sense?
- What do they mean?
- How are they useful?
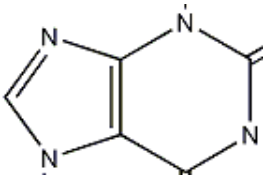
morphological info. and simple statistics

Semantic Information

Not all frequent patterns are useful, only meaningful ones …

Annotate patterns with semantic information

A Dictionary Analogy

Word: "pattern" – from Merriam-Webster

Non-semantic info.

Definitions indicating semantics

Synonyms

Related Words

Main Entry: ¹pat*tern*
Pronunciation: 'pa-tern
Function: *noun*
Etymology: Middle English *patron*, from Middle French, from Latin *patronus*
Date: 14th century

1 : a form
2 : somethi
pattern>
3 : a mode
casting
4 : an artist
5 : a natur

component
12 : freque
synonyms
   –pat*terne*

Main Entry: **pattern**
Function: *noun*

1
nonyms MODEL 2, archetype, beau ideal, ensample, example, exemplar, ideal, rror, paradigm, standard
Related Word original
   2
Synonyms FIGURE 3, design, device, motif, motive
Related Word patterning

onyms ORDER 8, method, orderliness, plan, system
Related Word arrangement, constellation

Semantic Analysis with Context Models

- Task1: Model the context of a frequent pattern

  Based on the Context Model...

- Task2: Extract strongest context indicators

- Task3: Extract representative transactions

- Task4: Extract semantically similar patterns

Annotating DBLP Co-authorship & Title Pattern

Database:

Authors	Title
X.Yan, P. Yu, **J. Han**	**Substructure** Similarity **Search** in Graph Databases
...	...
...	...

Frequent Patterns

P₁: { x_yan, j_han }

Frequent Itemset

P₂: "substructure search"

Semantic Annotations

Pattern	{ x_yan, j_han}
Non	Sup = …
CI	{p_yu}, graph pattern, …
Trans.	**gSpan: graph-base……**
SSPs	{ j_wang }, {j_han, p_yu}, …

Context Units

< **{ p_yu, j_han}**, **{ d_xin }**, … , **"graph pattern"**, … **"substructure similarity"**, … >

Pattern = {xifeng_yan, jiawei_han}

Annotation Results:

Context Indicator (CI)	graph; {philip_yu}; mine close; graph pattern; sequential pattern; …
Representative Transactions (Trans)	**> gSpan: graph-base substructure pattern mining;** **> mining close relational graph connect constraint; …**
Semantically Similar Patterns (SSP)	{jiawei_han, philip_yu}; {jian_pei, jiawei_han}; {jiong_yang, philip_yu, wei_wang}; …

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Summary

- Roadmap: Many aspects & extensions on pattern mining

- Mining patterns in multi-level, multi dimensional space

- Mining rare and negative patterns

- Constraint-based pattern mining

- Specialized methods for mining high-dimensional data and colossal patterns

- Mining compressed or approximate patterns

- Pattern exploration and understanding: Semantic annotation of frequent patterns

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